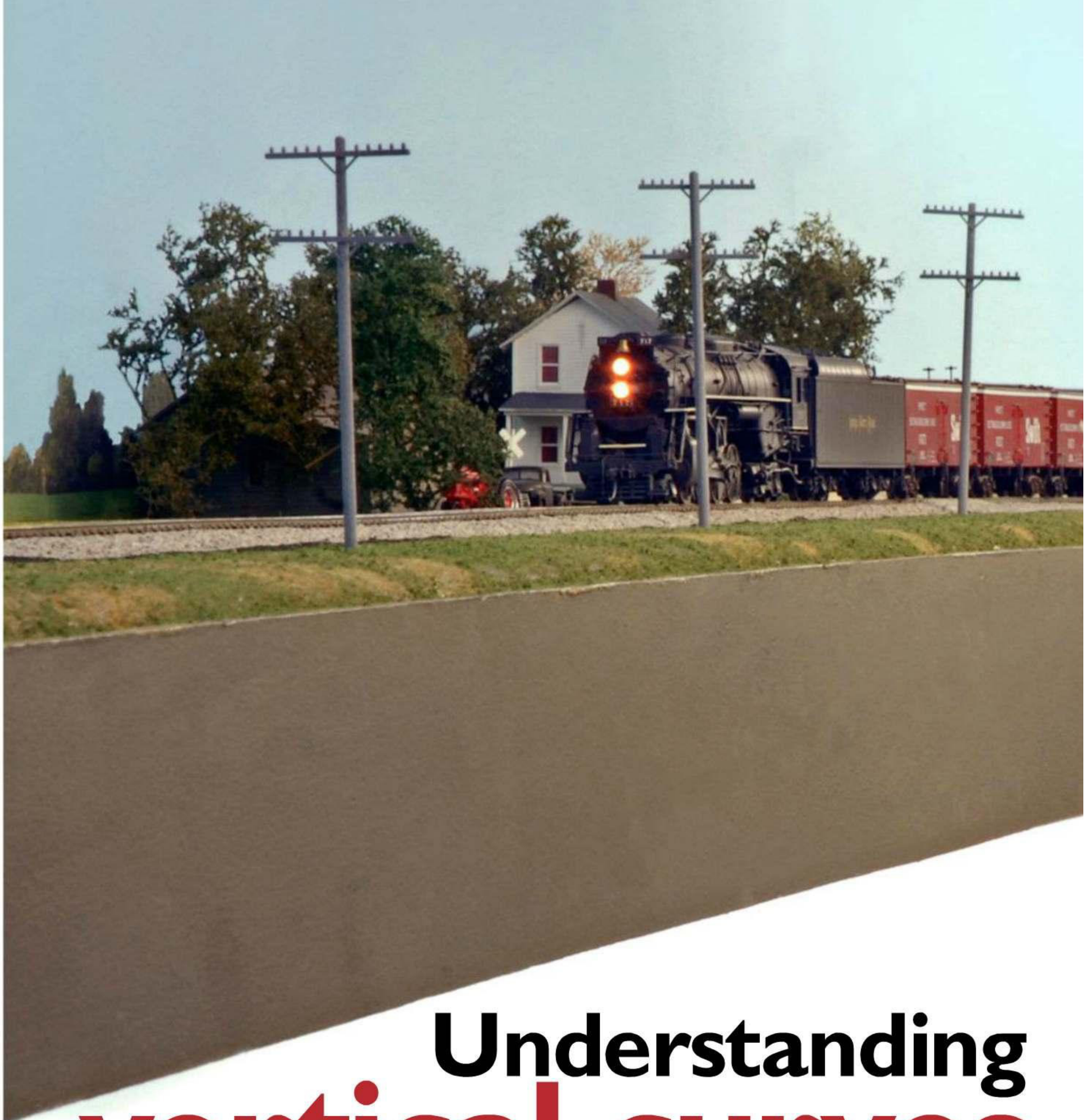
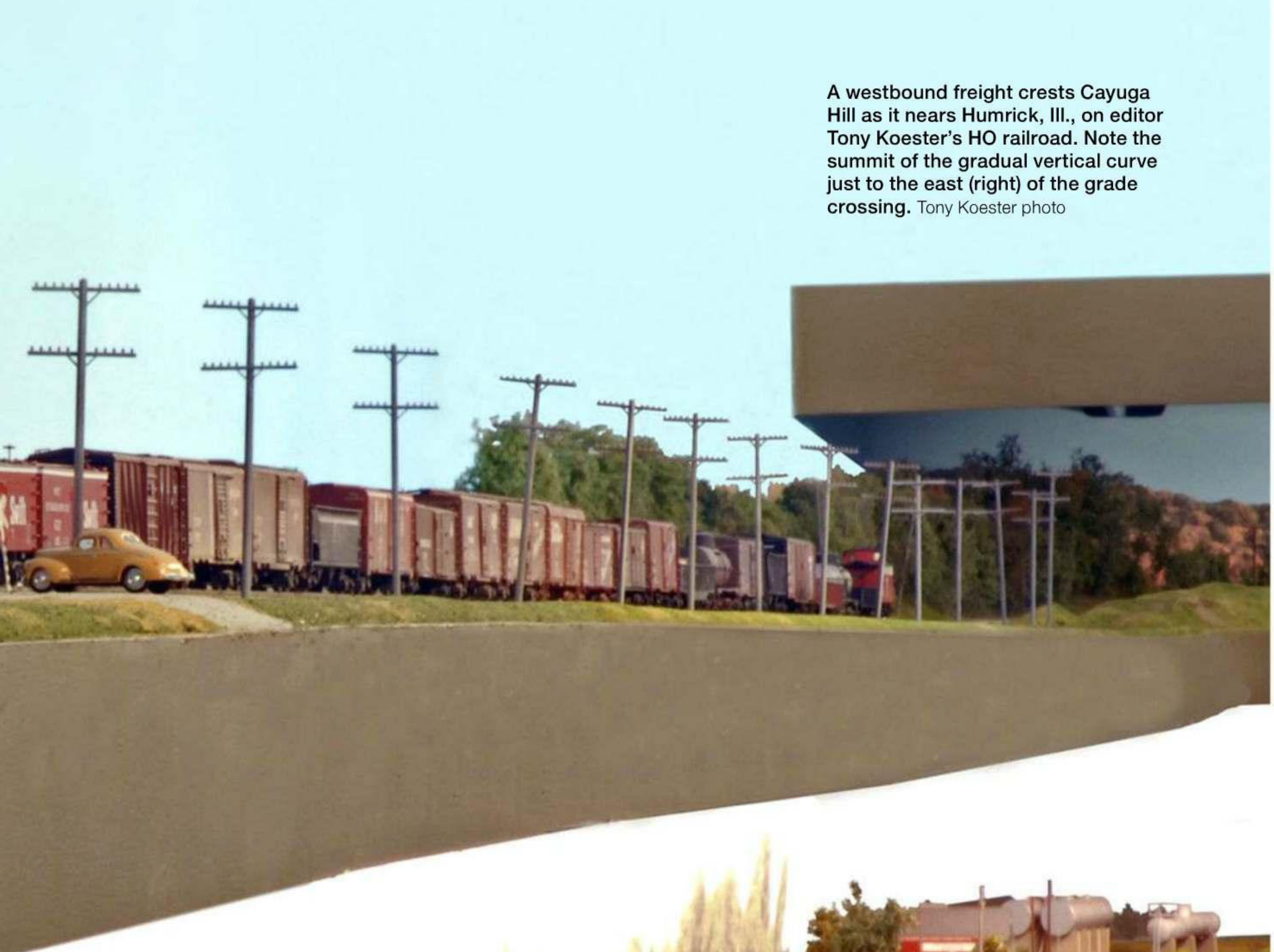


Understanding vertical curves

They're as important as the minimum-radius, horizontal type

By Van S. Fehr



A westbound freight crests Cayuga Hill as it nears Humrick, Ill., on editor Tony Koester's HO railroad. Note the summit of the gradual vertical curve just to the east (right) of the grade crossing. Tony Koester photo

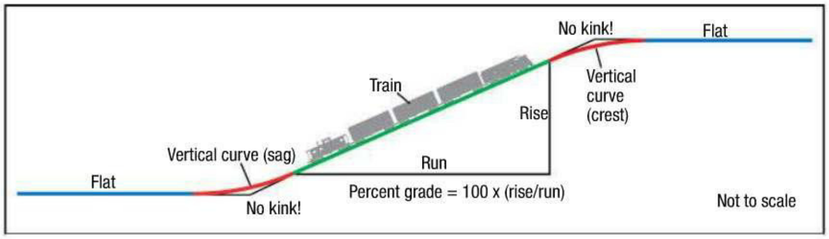


Fig. 1 Getting rid of the kinks. Grades require vertical curves at each end to avoid angular kinks and ensure reliable operation.

Prototype railroads use grades to accommodate changes in terrain and to get over or under obstacles. Railroads follow rivers, build bridges, bore tunnels, and use loops to keep grades to a minimum. Where grades begin or end, as in the photo on the previous page, civil engineers use a vertical curve to avoid the unacceptable kink that would otherwise occur.

Like full-size railroads, model railroads need to employ well-designed and -constructed vertical curves at each end of every grade for smooth, reliable operation. But a search for a simple method to design and construct reliable vertical curves was fruitless, so I developed a plan. It utilizes sound engineering principles and works for any scale. I'll describe the engineering study I did, the simple design formulas I developed using its results, and – happy days! – a math-less construction method. I'll close with an example showing how it all works.

Prototype vertical curves

I started by reviewing prototype practice for grades and vertical curves. Civil engineers quantify grade as the ratio of its rise (change in elevation) to its run (horizontal distance) multiplied by 100 to express grade as a percent. Because grade is a ratio, it does not depend on model scale.

The American Railway Engineering and Maintenance-of-Way Association (AREMA) provides a formula for the vertical curve horizontal length, given the two grades on each side of the vertical curve, a specified train speed, and a standard vertical acceleration. Engineers set the calculated length to a positive value, even if the arithmetic difference in the two grades produces a negative result.

A vertical transition curve has the shape of a parabola. An upward curving transition is a sag; one curving downward is a crest. These names apply whether or not there's a high or low spot somewhere between the ends. The net elevation is the difference between the end-point elevations.

For the typically small difference in grades, in both the prototype and the model, a circular arc is a close approximation of a parabola. In fact, the AREMA length formula actually comes from the physics of circular motion. The radius then depends only on the vertical curve length and the two adjacent grades. Though radius serves no purpose in construction, it's useful as a point of comparison.

Model vertical curves

Figure 1 shows a typical model railroad grade, exaggerated to show its features. Because model railroad cars have no brakes, modelers usually build yards, spurs, and other switching locations on a flat (zero grade) so cars standing alone won't roll away. When two such switching locations are at different elevations, a grade with vertical curves at each end connects them. The crest has the same shape as the sag, but it's upside down.

Applying prototype practice to a model railroad isn't simply a matter of scaling. Prototype vertical curves are many times the length of the longest rolling stock. For example, the AREMA formula shows that a 50-mph freight train transitioning from level to a 2 percent grade requires a vertical curve length of 1,075 feet, more than 10 times the length of the longest freight cars. The curve's radius exceeds 10 miles. The equivalent HO scale length is over 12 feet, likely longer than an entire grade. For practical use, the model vertical curve length and radius must be much smaller than the scaled prototype – another case of necessary selective compression.

Two MR articles suggested radius values: one by John Lukesh (March 1970) and another by Philip Page (November 1972). In the February 1974 *Model Railroader*, editor Linn Westcott asked for a volunteer to study a variety of equipment sizes to determine vertical curve tracking ability. No volunteers appeared in the years that followed.

So what should the radius and length be? I found an answer in the study I did – the one Linn Westcott requested 40 years ago.

Vertical curve radius study

While the value of a vertical curve radius isn't important for construction, it is for reliable operation. Sharp model railroad vertical curves can derail rolling stock and cause inadvertent uncoupling or short circuits if metal locomotive pilots touch the rails. To learn how to avoid these unpleasantries, I developed equations for the minimum vertical curve radius at which they occur, verified them by hand calculations, and programmed them into an Excel spreadsheet.

I selected 76 freight and passenger cars; steam and articulated steam locomotives; and diesel locomotives representing a broad range of equipment and era, and used their HO scale dimensions. The dimensions came from the *Model Railroader Cyclopedia: Vol. 1, Steam Locomotives* and *Vol. 2, Diesel Locomotives* (Kalmbach Publishing Co.). I scaled prototype car dimensions from a variety of printed and online sources.

I used flange depths from National Model Railroad Association (NMRA) Recommended Practice RP-25 and NMRA Standard S-4.1 (Proto 87.1). I measured Kadee HO standard and scale coupler knuckle height dimensions, and used a representative .05" coupler droop. For pilot clearance, I scaled 3 and 6 prototype inches, the extremes allowed by the Code of Federal Regulations, Title 49, Section 229.123. These parameters combine into 16 unique cases that I applied to each of the 76 equipment selections. This produced 1,216 calculated

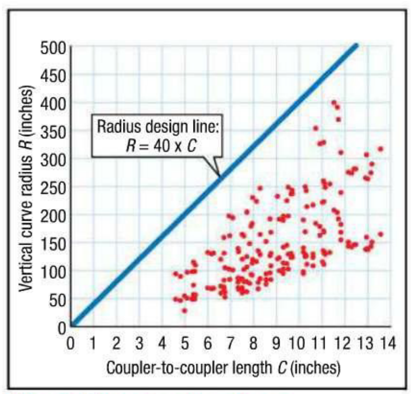


Fig. 2 Plotting the data. A formula sets the vertical curve radius for the given coupler-to-coupler length of a piece of rolling stock. The red circles represent the minimum vertical curve radius for each of the 1,216 combinations (some are duplicates).

minimum radius values, graphed versus rolling stock overall coupler-to-coupler length in **fig. 2**.

Showing considerable variation, the limiting condition for the minimum radius was typically flange exposure on a sag, but all of the other possible conditions were limiting for some equipment in some of the cases. Much of the variation is due to flange depth, the roughly upper-half of the points associated with Proto:87.1 flange depth, with some overlap with RP-25 flange depth results. The rest is due to the variations in pilot clearance, knuckle height, and coupler droop.

The results tend to converge at the origin, suggesting the vertical curve radius should be directly proportional to the coupler-to-coupler length. To include equipment not analyzed, I selected a proportionality factor of 40, producing the radius design line shown in **fig. 2**, opposite. Simple statistics show that over 99 percent of all rolling stock will have a minimum radius falling below this line. Even better, because the proportionality factor is a ratio, the value "40" applies to all model scales.

Great! A simple formula to calculate a vertical curve radius – it's 40 times the coupler-to-coupler length of any instance of rolling stock. Equally important, the minimum length of the vertical curve is always its radius times the adjacent grade. This leads to the length formula in "Grade and vertical curves" on the next page, and avoids difficulties in the methods of Lukesh and Page.

Vertical curve design

Model vertical curves must operate reliably and should look prototypical. The length formula on the next page ensures reliability, but prototypical appearance is another matter.

Appearance is subjective. I think the vertical curve length should never be less than the coupler-to-coupler length of the longest equipment traversing it, or it begins to look more like a roller coaster than a railroad. It's possible, with shallow mainline grades, that the calculated vertical curve length will be less than the coupler-to-coupler length. In that case, set the curve length to the coupler-to-coupler length or longer, longer being more prototypical. For steam engines, exclude the tender and use the coupler-to-drawbar distance.

Vertical curve length in excess of twice the coupler-to-coupler length, although looking more prototypical, may take too much space. However, some extra-long, modern-era equipment may require vertical curve lengths that exceed twice the coupler-to-coupler length. In that case, use the calculated length to ensure reliability.

Without the vertical curve at the crest of this grade in the staging area of *Model Railroader*'s club layout, couplers would separate at the kink. Bill Zuback photo

Figure 3 details the geometry (exaggerated) of a model railroad grade between two flat locations. Notice the actual grade (solid line) is steeper than the nominal grade (dashed line) obtained by dividing the overall height by the overall horizontal distance. It's better to start with the required grade and adjust the overall height or overall horizontal distance accordingly. If you don't do this, your final actual grades can be steeper than you expect, and locomotive performance may suffer intolerably.

Once you select a grade, measure the coupler-to-coupler length of the longest equipment you plan to operate on the grade. Then calculate the vertical curve length and the vertical curve net elevation using the formulas in the sidebar. As a fortuitous consequence of the "40" proportionality factor, for any grade at or below .025 (2.5 percent), simply set the vertical curve length to the coupler-to-coupler length (or longer). Calculate the vertical curve length only for grades exceeding .025 (2.5 percent).

Complete the design of the grade and its vertical curves using the formulas in the sidebar. Once you've set your required grade, make the compromises between the overall grade height and overall length using the last two formulas.

Vertical curve construction

My construction method requires no elevation calculations between the vertical curve end points. That's because the subroadbed itself "does the math" automatically.

Here's why. Subroadbed is normally a constant width and thickness along the vertical curve. When gently bent, it takes a natural, predictable mathematical shape. This behavior is what makes the flexible stick used in the "bent-stick" method work so well for laying out horizontal easements. Better, for vertical curves, when you hold one end flat, and hold the other end to the net elevation and slope of the adjacent grade, the subroadbed takes the shape of a parabola, just like the prototype.

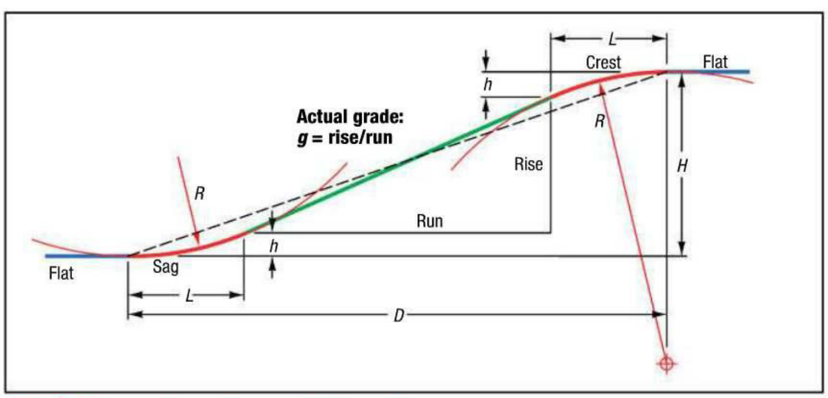


Fig. 3 Geometry of a model grade. Compromises in overall length and height will be necessary to ensure the actual grade (solid line) meets your goal.

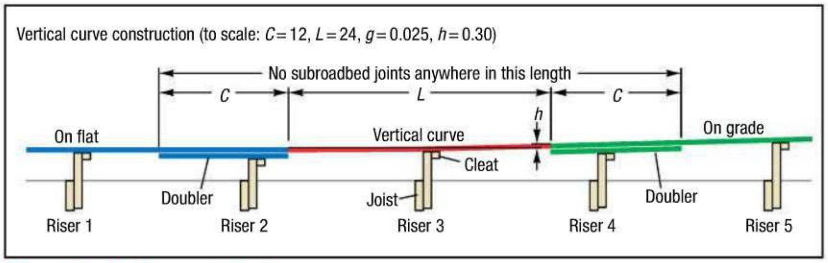


Fig. 4 Building vertical curves. Keeping in mind the location of joints and the stiffness of the subroadbed is important in constructing a vertical curve.

There are numerous documented methods for constant-grade construction. One method uses risers spaced at convenient intervals along the grade, adjusted vertically to hold the subroadbed to the desired grade. The example that follows designs such a grade with vertical curves at its ends, but only describes building one of the curves.

Grade design and construction

Suppose you wish to run HO scale passenger trains with 85-foot cars having an HO scale coupler-to-coupler distance C = 11.7". Your trains are just a few cars in length and you expect your somewhat shorter motive power can pull them up a desired 2.5 percent (.025) grade without difficulty. You have two flat towns separated vertically by 2" and horizontally by 80".

Because your desired grade is 2.5 percent, you see the minimum vertical curve length is simply 11.7", which you round up to 12" for simplicity. Wanting the curves to look more prototypical, you select twice that length so L = 24". Using the equation at right, you calculate the net elevation at the end of the vertical curve as h = .025 x 24 / 2 = .30".

So far, so good. You check the grade overall height by calculating H = .025 x (80 – 24) = 1.4", considerably less than the 2" you need. Not so good! You then calculate the overall length using your required overall height and get D = 2.0/.025 + 24 = 104", far beyond your initial horizontal distance. This is the dilemma we face when designing grades – we must often make compromises. You decide you can't increase the grade because locomotive performance will suffer. You could shorten the vertical curves to 12", but decide you would be unhappy with the appearance. You also decide the 2" elevation change is important. That means you have to accept the calculated overall length of 104" and increase the distance between the towns.

Figure 4, drawn to scale, illustrates one method of vertical curve construction, specifically for this example of an HO scale .025 (2.5 percent) mainline grade, a 24" vertical curve length, and a calculated .3" net elevation. The risers are 16" on center, attached to joists mounted on L-girders or open-grid benchwork, all shown as black lines. Blue lines represent the lower flat area, red the vertical curve (sag), and green the constant grade of the subroadbed. At the high end, the vertical curve (not shown) has the same geometry, but inverted, as **fig. 1** and **fig. 3** illustrate.

Allow no joints in the subroadbed in or near the vertical curves, or you could get an unacceptable kink at the joint. Glue and screw a doubler, the same width and thickness as the subroadbed, to the subroadbed along a distance C just outside each end of the vertical curve. The doublers make the subroadbed eight times stiffer, forcing it to bend only within the vertical curve.

First, fasten risers 1 and 2 to the joists and the subroadbed to the cleats, making sure the flat section stays level as you proceed. Then, by trial and error, adjust risers 4 and 5. Clamp riser 4 in place until the end of the vertical curve is the height h above the flat (.3" in this case). Measure the grade using an appropriate tool, such as Micro-Mark's miniature digital level.

At this point you may find the grade is slightly steeper than you want. Clamp the subroadbed to the cleat on riser 5. Push the subroadbed down until you achieve the desired grade, then clamp riser 5 to the joist. Check the vertical curve elevation at the end of the vertical curve, and again adjust riser 4 to match h. Adjust riser 5 again until the grade is correct. At this point the vertical curve end elevation should be close enough. Screw risers 4 and 5 and the subroadbed in place, and remove the clamps.

Finally, adjust riser 3 until it just touches the bottom of the subroadbed. Screw it to the joist. Then screw the subroadbed to the cleat, tightly enough to be secure, but without changing the shape of the vertical curve.

The thicker the subroadbed, the harder it is to bend into a vertical curve. Some modelers use ¾" plywood for subroadbed, which is more than three times stiffer than ½" plywood, and eight times stiffer than ⅜" plywood. Try bending a sample length of your subroadbed, and if you decide it's too difficult, consider reducing its thickness.

Ends justifying the means

It's difficult to embrace using analytical methods – math! – when we're designing and building a model railroad. But when slight errors can cause big problems later on, it's well worth the added mental effort. MRP

This is Van Fehr's second article in MRP. Van, who models in HO scale, is the NMRA Data Sheet Committee assistant chair and NMRA RP-12 Turnout Working Group chair.

Learning points

- For reliable operation in any scale, design grades and vertical curves using the given formulas.
- For good appearance, use vertical curves that are no shorter than the coupler-to-coupler length of the longest equipment traversing them, or up to twice as long if space permits.
- Take advantage of the flexibility of the subroadbed to aid in vertical curve construction, eliminating the need to calculate intermediate elevations.

Grade and vertical curves

Actual grade:
g = rise / run
percent g = 100 x rise / run

Vertical curve length:
L = 40 x g x C

If the calculated length L is less than the coupler-to-coupler distance C, set L = C (or longer) for good appearance.

Vertical curve net elevation:
h = g x L / 2

Grade overall height:
H = g x (D – L)

Grade overall distance:
D = H / g + L